In a nutshell: The golden-section search

Given a continuous real-valued function f of a real variable defined on the interval $[a_0, b_0]$ where there is known to be a minimum on that interval. We will assume that the minimum is not at an end-point. This algorithm uses iteration, bracketing and weighted averages to find the minimum.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the minimum cannot exceed this value.
ε_{abs}	The difference in the value of the function after successive steps cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$
2. Let $w \leftarrow b_k - a_k$, $x_1 \leftarrow b_k - w/\phi$, $x_2 \leftarrow a_k + w/\phi$.
3. Given that we have constrained the minimum to $[a_k, b_k]$, if $(b_k - a_k)/\phi < \varepsilon_{\text{step}}$ and $\min\{|f(x_1)|, |f(x_2)|\} < \varepsilon_{\text{abs}}$, we are done, and return whichever interior point has the smallest absolute value, returning either in the very unlikely case that $|f(x_1)| = |f(x_2)|$.
4. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
5. If $f(x_1) < f(x_2)$, let $a_{k+1} \leftarrow a_k$ and $b_{k+1} \leftarrow x_2$;
6. if $f(x_2) < f(x_1)$, let $a_{k+1} \leftarrow x_1$ and $b_{k+1} \leftarrow b_k$;
7. otherwise $f(x_2) = f(x_1)$, which is extremely unlikely, but in this case, we would have to apply the golden-section search to both sub-intervals to determine which sub-interval to continue searching.
8. Increment k and return to Step 2.

Convergence

Having found x_1 and x_2, the maximum error at any step is $(b_k - a_k)/\phi$, so with each step, the maximum error is reduced by $1/\phi$. Thus, if h is the error, the error decreases according to $O(h)$.